LECTURE NOTES

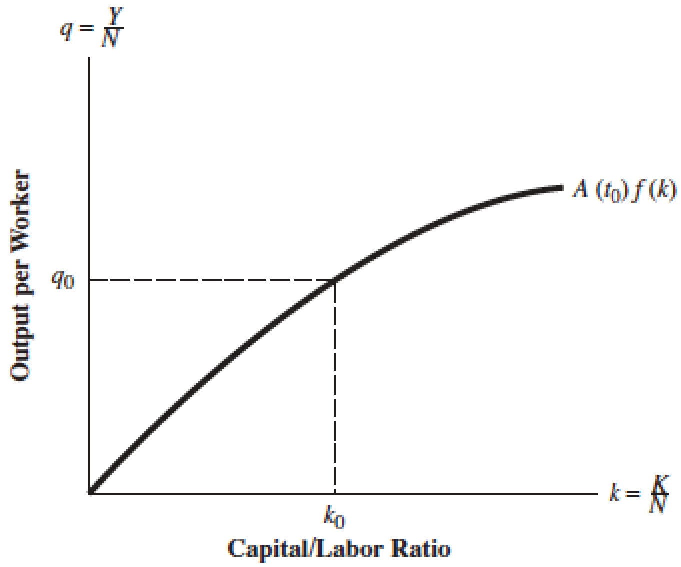
Chapter 20: Long-Run Economic Growth: Origins of the Wealth of Nations

- Adam Smith's (and the classics) question was primary about **wealth** (long-run), not about **cycles** (short-run)

1. The Neoclassical Growth Model

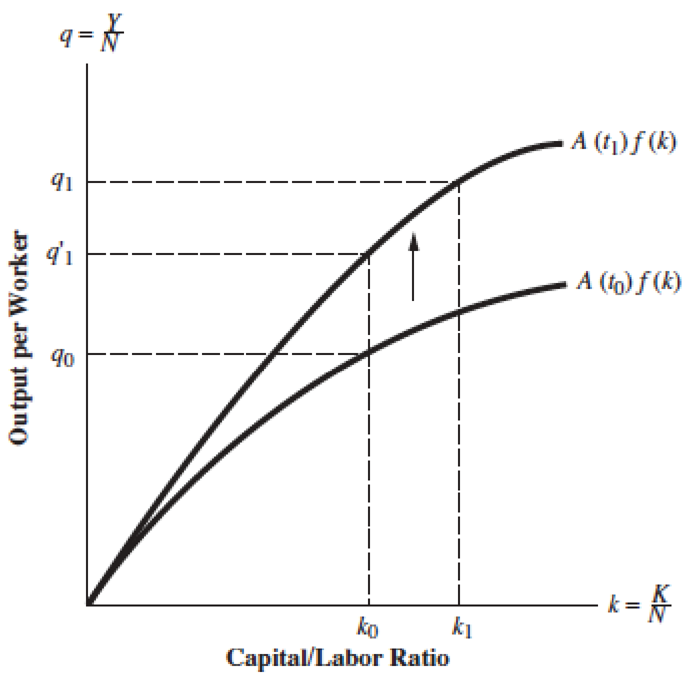
- Growth and the Aggregate Production Function
    - Review section 3 in lecture notes for chapter 3
    - $Y(t) = A(t) \cdot F\big(K(t), N(t)\big)$
    - Growth over time: $\frac{\dot{Y}}{Y} = \frac{\dot{A}}{A} + w_K \frac{\dot{K}}{K} + w_N \frac{\dot{N}}{N}$, $\;w_i = weights, i = \{K, N\}$
    - $\dot{Y} = \frac{\partial Y}{\partial t}$, $\dot{K} = \frac{\partial K}{\partial t}$, $\dot{Y} = \frac{\partial N}{\partial t}$
    - Assume constant returns to scale, then you can write: $q = \frac{Y}{AN} = f(k)$, $\;k = \frac{K}{AN}$
        - Assume: $Y(t) = A(t) \cdot \left[K(t)^{\gamma} \cdot N(t)^{(1-\gamma)}\right]$
        - Then: $\frac{Y(t)}{A(t) \cdot N(t)} = \frac{A(t) \cdot \left[K(t)^{g} N(t)^{1-\gamma}\right]}{A(t) \cdot N(t)} = \frac{A(t)}{A(t)}\left[\left(\frac{K(t)}{N(t)}\right)^{\gamma} \cdot \left(\frac{K(t)}{N(t)}\right)^{(1-\gamma)}\right] \rightarrow q(t) = f\big(k(t)\big)$

FIGURE 20-1 Aggregate Production Function: Equation (20.3)



The intensive form of the production function shows output per worker ($q = Y/N$) corresponding to each capital/labor ratio ($k = K/N$) for a given technology [$A(t_0)$]. As the capital/labor ratio rises, output per worker increases but at a declining rate, reflecting diminishing returns to increases in capital per worker.

FIGURE 20-2 Growth in Output per Worker



Output per worker increases from q_0 to q'_1 when, as the result of technological progress, the production function shifts upward from $A(t_0)f(k)$ to $A(t_1)f(k)$. There is a further increase in output per worker from q'_1 to q_1 as a result of an increase in the capital/labor ratio from k_0 to k_1.

- Sources of Growth in the Neoclassical Model: Solow's Model
  - Growth sources: (1) technology, (2) capital, and (3) labor
  - In this model savings (and therefore investment) does not affect long-run growth
  - Assume s is the <u>exogenous</u> savings rate
    - $I(t) = s \cdot Y\big(k(t)\big)$
  - Let d be the depreciation rate
    - $D(t) = d \cdot K(t)$
    - $\dot{K}(t) = s \cdot Y(t) - d \cdot K(t)$
    - $\dot{k}(t) = s \cdot f\big(k(t)\big) - (d + g_N + g_A) \cdot k(t)$
  - Because $K = 0 \rightarrow I(K = 0) = D(k = 0) = 0,\ \frac{\partial I(t)}{\partial k(t)} > 0, \frac{\partial I(t)^2}{\partial^2 k(t)} < 0, and\ \frac{\partial D(t)}{\partial K(t)} = d < \frac{\partial I(t)}{\partial k(t)}$
    - Then:

      $There\ \exists\ k^* > 0\ s.t.\ \dot{k} = 0 \rightarrow s \cdot f(k(t)^*) = (d + g_N + g_A) \cdot k(t)^*\ [steady\ state]$
    - Then, in the long-run growth cannot be achieved through capital accumulation
    - A change in s affects the *level* of wealth, but <u>not the growth rate in the long-run</u>
    - $q(t) = f\big(k(t)\big)$
    - $A(t) \cdot \dot{q} = \dot{A}(t) \cdot \underbrace{f(k(t)^*)}_{q(t)} + A(t) \cdot \underbrace{f\big(\dot{k}(t)^*\big)}_{0}$
    - $\frac{\dot{q}(t)}{q(t)} = \frac{\dot{A}(t)}{A(t)} + 0 \rightarrow g_q = g_A$
  - In the long-run, the growth of output per capita is the growth rate of technology (TFP: Total Factor Productivity)
  - Solow's model does not assume that consumption is maximized
    - Because the savings rate, the saving rate that maximizes consumption (golden rule) may be different from the exogenous savings rate
  - Growth Accounting
    - $Y = A \cdot F(K, N)$
    - $\partial Y = \frac{\partial Y}{\partial A} \cdot \partial A + \frac{\partial Y}{\partial K} \cdot \partial K + \frac{\partial Y}{\partial N} \cdot \partial N$
    - $\frac{\partial Y}{Y} = \frac{\partial Y}{\partial A} \cdot \frac{\partial A}{Y} + \frac{\partial Y}{\partial K} \cdot \frac{\partial K}{Y} + \frac{\partial Y}{\partial N} \cdot \frac{\partial N}{Y}$
    - $\frac{\partial Y}{Y} = \frac{\partial Y}{\partial A} \cdot \frac{\partial A}{A} \cdot \frac{A}{Y} + \frac{\partial Y}{\partial K} \cdot \frac{\partial K}{K} \cdot \frac{K}{Y} + \frac{\partial Y}{\partial N} \cdot \frac{\partial N}{N} \cdot \frac{N}{Y}$
    - $\frac{\partial Y}{Y} = \left(\frac{\partial Y}{\partial A} \cdot \frac{A}{Y}\right) \cdot \frac{\partial A}{A} + \left(\frac{\partial Y}{\partial K} \cdot \frac{K}{Y}\right) \cdot \frac{\partial K}{K} + \left(\frac{\partial Y}{\partial N} \cdot \frac{N}{Y}\right) \cdot \frac{\partial N}{N}$
    - $\frac{\partial Y}{Y} = \varepsilon_A \cdot g_A + \varepsilon_K \cdot g_K + \varepsilon_N \cdot g_N;\quad \frac{\partial Y}{\partial A} \cdot \frac{A}{Y} = \frac{F(K,N) \cdot A}{Y} \rightarrow \varepsilon_A = 1$
    - $\frac{\partial Y}{Y} = g_A + \varepsilon_K \cdot g_K + \varepsilon_N \cdot g_N$
  - Solow's residual or TFP: Growth of output that cannot be explained by factors of production
    - $g_A = g_Y - (\varepsilon_K \cdot g_K + \varepsilon_N \cdot g_N)$

2. Recent Developments in the Theory of Economic Growth

- Solow Model says that growth depend on *exogenous* variables
- That's the same than assuming long-run growth, it does not explain it
- New Growth Theory: Make growth endogenous by building over Solow Model (rather than discard it)
- Endogenous Growth Models
    - Endogenous technological change
        - Share α of capital and labor is assigned to research and development (technology)
        - Solow: $\dot{A}$ is given
        - Endogenous technological change: $\dot{A} = G\left(\alpha_K \cdot K(t); \alpha_N \cdot N(t); A(t)\right)$
    - Human capital and learning by doing
        - Labor is affected by an efficiency coefficient $e > 1$
        - This decreases the diminishing marginal returns to capital
        - If capital has constant returns to scale, then the model results in long-run positive growth rates
        - Let H be physical and human capital
        - Then: $Y = A \cdot H$ where *H* has constant returns to scale
        - $q = Ah$
        - Then: $\dot{h}(t) = s \cdot f\left(K(t)\right) - (g_N + d) \cdot h(t)$
        - $g_h = s \cdot A - (g_N + d)$
        - $q(t) = Ah$
        - $\dot{q}(t) = \dot{A} \cdot h + A \cdot \dot{h}$
        - $g_q = g_A + g_h$

3. Intercountry Income Differences

- Solow Model predicts that countries that are farther away from the *steady state* experience higher growth rates
- Then, we should observe convergence in the level of GDP per cápita
- However, countries have <u>different</u> steady states
    - Why North Korea does not converge to Sour Korea’s GDP per capita?
    - Same history, institutions, culture
    - But different institutions
- Conditional convergence
    - The predicted convergence is conditional to the institutional framework of each country that defines “where” the steady state is located